

## Three-Dimensional Space 2

### Stage 3 Outcome

A student:

- › describes and represents mathematical situations in a variety of ways using mathematical terminology and some conventions MA3-1WMM
- › identifies three-dimensional objects, including prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views MA3-14MG

**Language:** Students should be able to communicate using the following language: object, shape, three-dimensional object (3D object), prism, cube, pyramid, base, uniform cross-section, face, edge, vertex (vertices), top view, front view, side view, net.

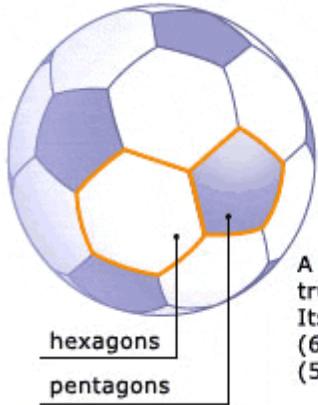
Teaching and Learning Activities	Notes/ Future Directions/Evaluation	Date/ LAC Icons
<p><b><u>Ignition Activities</u></b> <b>Barrier Game.</b></p> <p>Students work in pairs, with a barrier e.g. a folder or school bag between them. One student makes a three-dimensional model using construction materials. The student then instructs the partner to make a similar model. When complete, the models are compared and discussed.</p>		 Literacy
<p>Students create prisms and pyramids using a variety of materials, eg plasticine, paper or cardboard nets, connecting cubes. Sketch front, top and side views. Students describe to a partner how they constructed and drew their 3D object.</p>		 Literacy
<p><b>Nets – 3D Models</b> <a href="http://www.senteacher.org/worksheet/12/3D.html">http://www.senteacher.org/worksheet/12/3D.html</a></p> <p>Using a variety of nets, students create 3D models of prisms and pyramids. Cube, cuboid, pyramid, tetrahedron, pentagonal prism, pentagonal pyramid, cone, octahedron, rhombic prism, dodecahedron, cylinder, triangular prism, icosahedron, hexagonal pyramid, hexagonal prism.</p>		
<p><b>Illuminations – Fishing for the best prism</b> <a href="http://illuminations.nctm.org/LessonDetail.aspx?ID=L793">http://illuminations.nctm.org/LessonDetail.aspx?ID=L793</a></p> <p>In this lesson, students use polydrons to create nets of rectangular prisms. They discover that there are many configurations for rectangular prisms with the same volume, and determine that certain configurations minimize surface area. The lesson continues in a discovery activity related to building the most cost-efficient and appealing fish tank.</p> <div style="text-align: right; margin-top: 10px;">  </div>		 Literacy

**Geometry – 3D Shapes Interactive**

[http://www.learner.org/interactives/geometry/3d\\_prisms.html](http://www.learner.org/interactives/geometry/3d_prisms.html)

We live in a three-dimensional world. Every object you can see or touch has three dimensions that can be measured: **length**, **width**, and **height**. The room you are sitting in can be described by these three dimensions. The monitor you're looking at has these three dimensions. Even you can be described by these three dimensions. In fact, the clothes you are wearing were made specifically for a person with your dimensions.

In the world around us, there are many three-dimensional geometric shapes. In these lessons, you'll learn about some of them. You'll learn some of the terminology used to describe them, how to calculate their surface area and volume, as well as a lot about their mathematical properties.



A soccer ball is a truncated icosahedron. Its faces are hexagons (6 sides) and pentagons (5 sides).

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**Geometry – Nets of Solids**

<http://www.onlinemathlearning.com/geometry-nets.html>

**How many ways? Rectangular Prisms**

Students are given 24 interlocking cubes. They are asked to make a rectangular prism with a volume of 24 cubic units. They describe their rectangular prism in terms of its length, breadth and height and record this information. The teacher poses the question. 'Can you make other rectangular prisms with a volume of 24 cubic units?' Students attempt to do this, record the results and describe what they notice. Students draw a simple perspective drawing of each prism showing depth.

Variation: Students make prisms with a variety of volumes and discuss.

*DIFFERENTIATION - Extension*

- Ask students to suggest possible dimensions for a rectangular prism that has a volume of  $42 \text{ cm}^3$  without using snap cubes.

Students are challenged to create all the possible nets for a cube. Students could use polydrons, grid paper or tiles to create the nets. Students are encouraged to decide if

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<p>each solution is different or if it is the same net in a different orientation. Students record the nets on paper or by using a computer package.  <i>Variation:</i> Students draw the nets of other prisms and pyramids. They find nets of other three-dimensional objects.</p>		
<p><b>COMMON MISCONCEPTIONS:</b>  When filling a 3-D figure, students may think there can be gaps or overlaps with the cubes filling the object. Have students compare this to finding the capacity of a container. If you put an object in that container, you would displace space to be filled with liquid, and you would get an inaccurate measure of the capacity. Same goes for a solid figure-there can be no displacement (unfilled space) if you want an accurate measurement.</p> <p><b>ESSENTIAL QUESTIONS</b></p> <ul style="list-style-type: none"> <li>• What is the relationship between the size of the box and the number of cubes it will hold?</li> <li>• How does the volume change as the dimensions of the box change?</li> </ul> <p><b>MATERIALS</b></p> <ul style="list-style-type: none"> <li>• cube nets, scissors, tape, cm cubes, ruler, recording sheet</li> </ul> <p><b>GROUPING</b>  Individual/Partners</p> <p><b>TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION</b>  In this task, students will create boxes and discover how volume is related to the length, width, and height of cubes.  Comments: To introduce this task, show the cube net and ask this question? What could be done to this net so that the top of the cube will be open? Students should discern that the top square could be cut off. Tell students that they will be building open cubes of different sizes and filling them with cubes. Explain that they will need to measure the dimensions of each cube to complete the chart.  Once students have completed the task, lead a class discussion about the patterns they noticed. Allow students to explain their findings and any relationships they noticed. Also, allow students to share their conclusions about the relationships between volume and the dimensions of cubes. Finally, allow students to write about their findings in their math journals.</p> <p><b>Task Directions:</b> Using the cube net, have students construct cubes of different dimensions and fill them with cm cubes. Have them measure the dimensions and record them in the appropriate boxes on the recording sheet. Then they will count the number of cubes it took to fill the cube and record the volume of each cube. Have students discuss their findings to generalise statements about the relationship between the dimensions of the cubes and their volume.</p>		 Critical and creative thinking

**FORMATIVE ASSESSMENT QUESTIONS**

- What do you notice about the size of the open cubes and the number of cm cubes they can hold? Explain your thinking.
  - Could you predict how many cm cubes a container can hold, based on its measurements? Justify your answer.
  - Students may create their own cubes using grid paper to create nets.
  - Students may present a demonstration on drawing nets for cubes to the class.
- Intervention:

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**Exploring With Boxes**

**Materials:** cube nets, scissors, tape, cm ruler, cm cubes, recording sheet



**Directions:**

1. Work with a partner. Cut out the nets for the open cubes, fold up the sides, and tape them together.
2. Measure each open cube and record your findings in the chart below.
3. Fill each box (open cube) with cm cubes and count them to find the volume.
4. Record your findings in the chart below.
5. Write in your math journal and describe how the size of the box is related to its volume.

Box (Open Cube)	Length of Base	Width of Base	Height of Cube	Volume
A				
B				
C				

Findings \_\_\_\_\_  
\_\_\_\_\_

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**Geometric Skeletons**

Materials: tooth picks, gumdrops (or raisins, currants, miniature marshmallows), 3-D solid of a triangular prism

**Activity: Part One**

Show students the toothpicks and gumdrops, and tell them they will use these

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materials to make three-dimensional figures. Now, show them the 3-D solid of a triangular prism. Ask:

- What 3-D solid am I holding?
- What can you tell me about it?
- What are its characteristics?

Explain to students that they will be making skeletons of triangular prisms. Explain that for their skeletons, they will use toothpicks to show each edge where faces meet. They will also use gumdrops to show each vertex where edges meet. Ask:

- How could we find out how many toothpicks we need to make a skeleton of a triangular prism?
- How many gumdrops do you think it will take to make a triangular prism?
- Why do you think so?

Show students how to use the toothpicks and gumdrops to make a triangular prism.

Then, have them use the same materials to make their own triangular prisms.

Once they have completed them, discuss students' triangular prisms. Ask:

- What do the gumdrops represent? (vertices)
- What do the toothpicks represent? (edges)

Explain to students that the model they just made is called a skeleton (as opposed to a 3-D solid) of a triangular prism, because it only shows the edges and vertices of the shape. It does not show the faces. Remove one of the toothpicks from the prism, and then ask:

- Is this a skeleton of a triangular prism?
- Why or why not? (no, because it is now incomplete)

Divide the class into pairs of students, and distribute copies of Activity Sheet A as well as more toothpicks and gumdrops. Tell students to use the number of gumdrops and toothpicks shown on the chart to try to make skeletons of triangular and rectangular prisms. If they can make a prism with a given number of gumdrops and toothpicks, have them record "yes" in the fourth column of the chart.

If they cannot make a prism, have them record "no" in the fourth column. When their answer is "yes" in the fourth column, ask students to record, in the fifth column, whether the prism they built is a triangular or a rectangular one.

Finally, have them complete the last part of the activity sheet.

#### **Activity Sheet A**

Directions to students:

Use the number of gumdrops and toothpicks shown on the chart to try to make skeletons of triangular and rectangular prisms. If you can make a prism with a given number of gumdrops and toothpicks, record "yes" in the fourth column of the chart. If you cannot make a prism, record "no" in the fourth column. When your answer is "yes" in the fourth column, record, in the fifth column, whether the prism you built is a triangular or a rectangular one. Complete the last part of the activity sheet.

#### **Activity: Part Two**

Once students have completed the previous activity and their charts, show them

Activity Sheet A. Ask:

- Which of the skeletons that you built are prisms?
- What do you notice about the number of edges on the prisms?
- What do you notice about the number of vertices on the prisms?
- What is the relationship between the number of edges and the number of vertices?
- Which of the skeletons are not prisms? Why not?

Have students use their completed skeletons to help them explain their findings.

**Problem Solving**

Use gumdrops and toothpicks to build a prism.

Can you build a prism that has exactly 27 edges? Why or why not? Use drawings to show how you know.

Activity Sheet A:

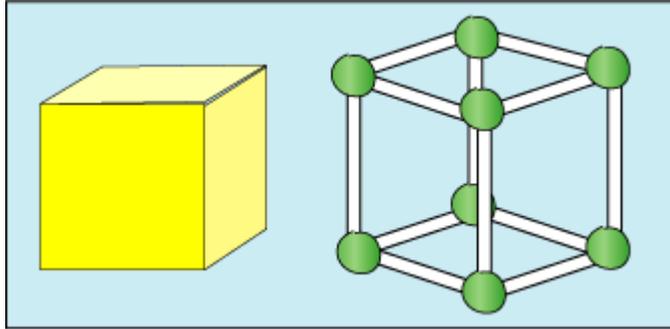
### Making 3-D Skeletons

	Gumdrops	Toothpicks	Is it Possible to Make a Prism? (yes/no)	Triangular or Rectangular Prism?
A	5	8		
B	6	9		
C	10	3		
D	6	10		
E	8	12		
F	10	15		
G	8	5		
H	12	18		
I	7	12		
J	5	9		

Discuss results.

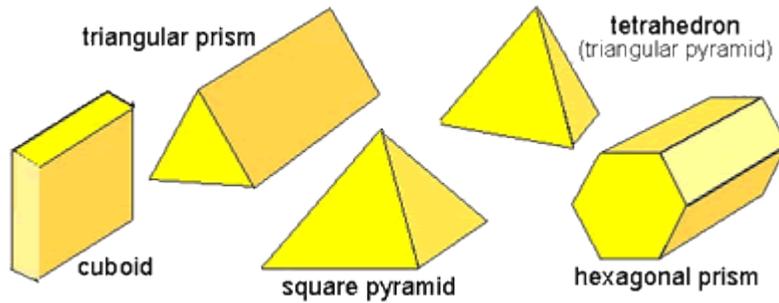
### Skeleton Shapes

Skeleton shapes are made with balls of modelling clay and straws.  
This shows a cube and a skeleton cube:



Students make their own Skeleton shapes . Pose questions such as; How many balls of modelling clay and how many straws does it take to make the cube?

Look at the shapes below and decide which piles are needed to make a skeleton of each shape.



### Making 3D Models

Provide students with top, side, and front views of a 3D shape. Ask them to construct a 3D model of that shape, given the different views. Discuss and pose questions.

## NAPLAN Teaching Strategies

[http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2010/index.php?id=numeracy/nn\\_spac/nn\\_spac\\_s3a\\_10](http://www.schools.nsw.edu.au/learning/7-12assessments/naplan/teachstrategies/yr2010/index.php?id=numeracy/nn_spac/nn_spac_s3a_10)

### Activity 2

Students use the *Surface Area, Volume and Nets* Learning Object to explore surface area, volume, 3D objects and nets. Objects include rectangular and triangular prisms; rectangular and triangular pyramids; cylinders and cones. Included are print activities, solutions and learning strategies.

[http://www.learnberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object\\_interactives/surfaceArea/use\\_it.html](http://www.learnberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object_interactives/surfaceArea/use_it.html)

